Lecture 12-2

Recursion

Binary Search: recursive implementation

```
/** Return the index of x in the sorted array a, or -1 if not found */
public static int find(int x, int[] a) {
    return find(x, a, 0, a.length - 1);
}

private static int find(int x, int[] a, int low, int high) {
    if (low > high)
        return -1;
    int med = (low + high) / 2;
    if (x == a[med])
        return med;
    else
        if (x < a[med])
            return find(x, low, med - 1);
        else
            return find(x, med + 1, high);
}
```

Integer.toString()

The task:

> Convert integer values to strings.
>
> For example, given the integer input 513, output the string "513"

Recursive thinking:

- Base case: If `n < 10` output "n"
- Reduction: an integer of the form `n = xyyy … y` (each x and y being a single digit) can be reduced using `x = n/10` and `yyy … y = n%10`
- Assembly: the concatenation operator `+` can be used to combine partial results into the final return value.

Integer.toString(): recursive implementation

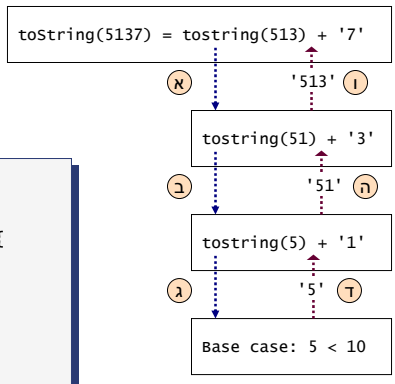
The task:

> Convert integer values to strings.
>
> For example, given the integer input 513, output the string "513"

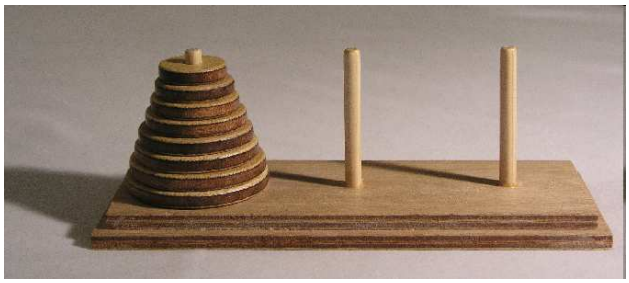
```
public class PrintIntegerDemo {

    public static void main(String[] args) {
        System.out.print(toString(513));
    }

    static String toString (int n) {
        if (n < 10)
            return "" + n;
        else
            return toString(n / 10) + n % 10;
    }
}
```



Tower of Hanoi



The rules of the game:

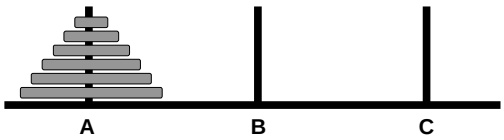
Move all the disks to another spindle, so that:

(1) only one disk moves at a time

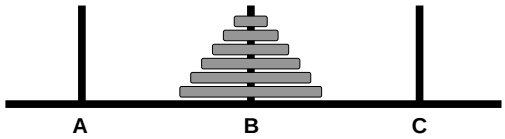
(2) The smaller disks are always at the top

Tower of Hanoi

Start:



Finish:



The rules of the game:

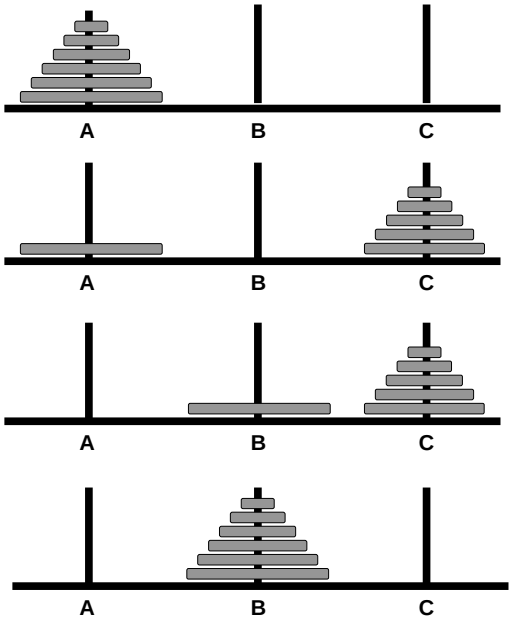
Move all the disks to another spindle, so that:

(1) only one disk moves at a time

(2) The smaller disks are always at the top

The recursive strategy

Start:



Finish:

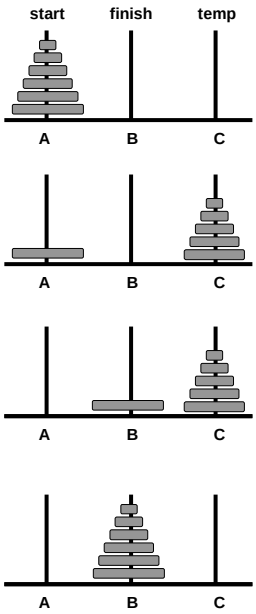
Move the top n-1 disks from A to C

Move the remaining disk from A to B

Move all the disks from C to B

Done!

Tower of Hanoi implementation



if n = 1 : move this single disk from start to finish

if n > 1 :

1. Move the top n-1 disks from start to temp, using finish
2. Move the bottom disk from start to finish
3. Move the top n-1 disks from temp to finish, using start

```
public class TowerOfHanoi {
  public static void main(String[] args) {
    moveTower(3, "A", "B", "C");
  }

  // Moves a tower of disks from A to B using C
  static void moveTower(int n, String start,
                        String finish, String temp) {
    if (n==1)
      System.out.println(start + "->" + finish);
    else {
      moveTower(n-1, start, temp, finish);
      System.out.println(start + "->" + finish);
      moveTower(n-1, temp, finish , start);
    }
  }
}
```

```
C:\WINDOWS\system3
D:\demo>java Tower
A->B
A->C
B->C
A->B
C->A
C->B
A->B
```

Simulation

```
moveTower(3, “A”, “B”, “C”)
```

Goal: move a tower of size 3 from A to B using C:

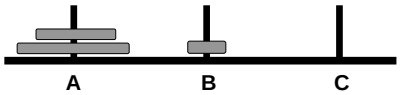
Goal: move a tower of size 2 from A to C using B:

Goal: move a tower of size 1 from A to B using C:

n	start	finish	temp
1	"A"	“B"	“C"

Things to do:

```
moveTower(n-1, start, temp, finish);
System.out.println(start + "->" + finish);
moveTower(n-1, temp, finish , start);
```



Simulation

```
moveTower(3, “A”, “B”, “C”)
```

Goal: move a tower of size 3 from A to B using C:

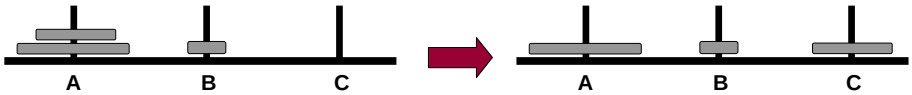
Goal: move a tower of size 2 from A to C using B:

n	start	finish	temp
2	"A"	“C"	“B"

Things to do:

~~moveTower(n-1, start, temp, finish);~~

```
System.out.println(start + "->" + finish);
moveTower(n-1, temp, finish , start);
```



Simulation

```
moveTower(3, "A", "B", "C")
```

Goal: move a tower of size 3 from A to B using C:

Goal: move a tower of size 2 from A to C using B:

n	start	finish	temp
2	"A"	"C"	"B"

Things to do:

~~moveTower(n-1, start, temp, finish);~~

~~System.out.println(start + " >" + finish);~~

```
moveTower(n-1, temp, finish , start);
```

Simulation

```
moveTower(3, "A", "B", "C")
```

Goal: move a tower of size 3 from A to B using C:

Goal: move a tower of size 2 from A to C using B:

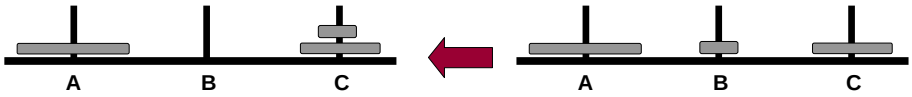
n	start	finish	temp
2	"A"	"C"	"B"

Things to do:

~~moveTower(n-1, start, temp, finish);~~

~~System.out.println(start + " >" + finish);~~

```
moveTower(n-1, temp, finish , start);
```



Permutations

The task:

Write a method `listPermutations(String s)` that generates a complete set of all the permutations of the characters in `s`.

For example, `listPermutations("abcd")` will generate:

```
abcd    bacd    cabd    dabc
abdc    badc    cadb    dacb
acbd    bcad    cbad    dbac
acdb    bcda    cbda    dbca
adbc    bdac    cdab    dcab
adcb    bdca    cdba    dcba
```

(one after the other)

Observations:

- If `length(s) = n`, there are `n * (n-1) * (n-2) * … * 1 = n!` permutations
- the list of permutations starting with "a" contains the prefix "a" followed by all the permutations of "bcd"

Strategy

Given "abcd" we have to generate:

```
abcd    bacd    cabd    dabc
abdc    badc    cadb    dacb
acbd    bcad    cbad    dbac
acdb    bcda    cbda    dbca
adbc    bdac    cdab    dcab
adcb    bdca    cdba    dcba
```

Print `'a'` + all permutations of `"abcd"` without charAt `0`

Print `'b'` + all permutations of `"abcd"` without charAt `1`

Print `'c'` + all permutations of `"abcd"` without charAt `2`

Print `'d'` + all permutations of `"abcd"` without charAt `3`


```
// String s without charAt(i):
rest = s.substring(0,i) + s.substring(i + 1 , s.length());
```

Implementation

```java
public class ListPermutationsDemo {

  public static void main(String[] args) {
    listPermutations("abcd");
  }

  public static void listPermutations(String s) {
    listPermutations("", s);
  }

  private static void listPermutations (String prefix, String s) {
      if (s.length() == 0)
          System.out.println(prefix);
      else
          for (int i = 0; i < s.length(); i++) {
              char ch = s.charAt(i);
              String rest = s.substring(0,i) + s.substring(i+1);
              listPermutations(prefix + ch, rest);
          }
      }
}
```

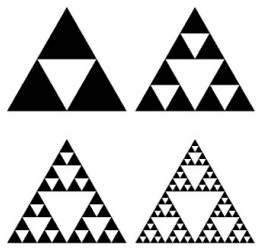
```
C:\WINDOWS\system32\c

D:\demo>java ListPerm
abcd
abdc
acbd
acdb
adbc
adcb
bacd
badc
bcad
bcda
bdac
bdca
cabd
cadb
cbad
cbda
cdab
cdba
dabc
dacb
dbac
dbca
dcab
dcba
```

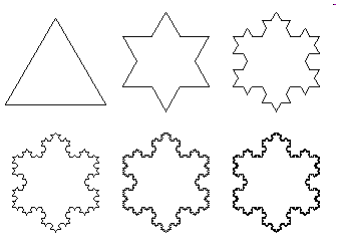

Fractals

Fractal: A geometric shape that can be split into parts, each of which is (either exactly or approximately) a reduced-size copy of the whole

Some well-known fractal shapes:

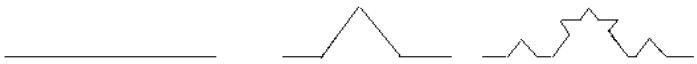


Sierpinski triangles



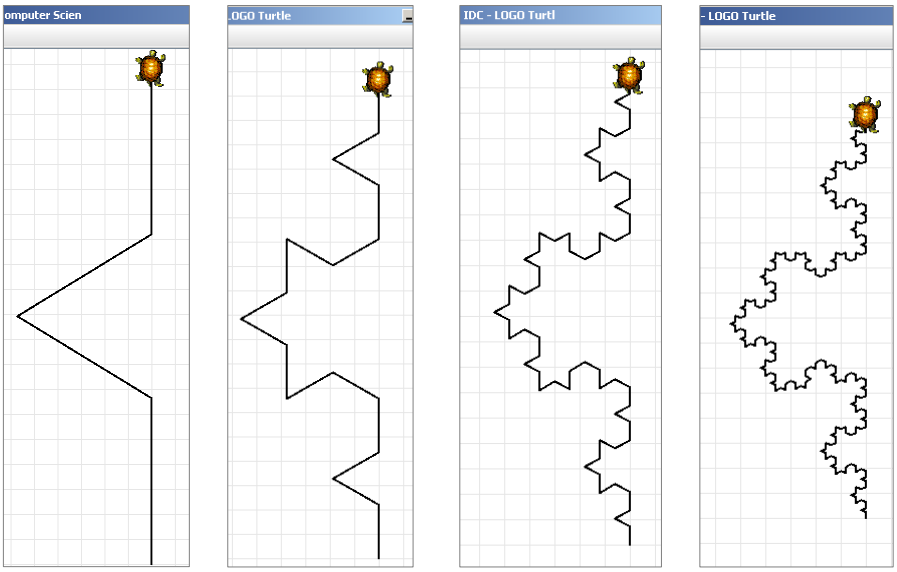
Koch snowflake

Typical generative strategy:



Level 0 Level 1 Level 2

TurtleFractal

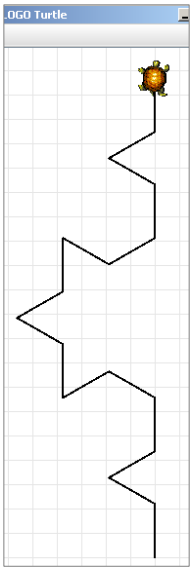


DrawFractal(500 , 1) DrawFractal(500 , 2) DrawFractal(500 , 3) DrawFractal(500 , 4)

Implementation

```
public class TurtleFractal {
  static Turtle turtle = new Turtle();
  public static void main(String[] args) {
      turtle.tailDown();
      drawFractal(500,2);
  }

  public static void drawFractal(int length, int level) {
      if (level==0)
          turtle.moveForward(length);
      else {
         drawFractal(length/3, level-1);
         turtle.turnLeft(60);
         drawFractal(length/3, level-1);
         turtle.turnRight(120);
         drawFractal(length/3, level-1);
         turtle.turnLeft(60);
         drawFractal(length/3, level-1);
      }
  }
}
```



Fractals